

# Multi Cumulative Prospect Theory and the Demand for Cliquet-Style Guarantees

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## Abstract

Expected Utility Theory (EUT) and Cumulative Prospect Theory (CPT) face problems explaining preferences of long-term investors. Previous research motivates that the subjective utility of a long-term investment also depends on interim value changes. Therefore we propose an approach that we call Multi Cumulative Prospect Theory. It is based on CPT and considers annual changes in the contract values. As a first application we can show that in contrast to EUT and CPT, this approach is able to explain the demand for guaranteed products with lock-in features, which in this framework generate a higher subjective utility than products without or with simpler guarantees.

**Keywords:** Behavioral Insurance, Prospect Theory, Guaranteed Products

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## 1 Introduction

Cumulative Prospect Theory (CPT), introduced by Tversky & Kahneman (1992), has become one of the most prominent behavioral theories in finance, especially as a behavioral counterpart to Expected Utility Theory (EUT). This is due to the fact that CPT can explain behavior that can not be explained by EUT, but is still frequently observed in real life. While complex financial products and long-term investments are well studied under EUT, an analysis of such products under CPT has only recently been in the focus of academic literature. Døskeland & Nordahl (2008) consider different participating life insurance contracts under CPT to explain the demand for guaranteed products. In an empirical work, Dierkes *et al.* (2010) investigate the preferences of a CPT investor considering different investment strategies and time horizons. Historical and Monte Carlo simulations were used by Dichtl & Drobetz (2011) to analyze portfolio insurance strategies based on simple CPPI (Constant Proportional Portfolio Insurance) strategies. Ebert *et al.* (2012) determine the “optimal” specification of different guarantee types, where optimality is defined as creating the maximum subjective utility for a CPT investor. A main result of these papers is that, in contrast to EUT, CPT can explain the demand for guarantees.<sup>1</sup> Nevertheless, even CPT is not able to explain the popularity of more complex guaranteed products, such as ratchet or cliquet guarantees, since a CPT investor should prefer a simple guarantee at maturity over more complex guarantees with lock-in features.

In both, EUT and CPT, the preferences of the investor only depend on the distribution of one terminal value. In reality, however, investors tend to re-evaluate a financial product regularly, e.g. annually when they receive a financial statement. Information about a good performance in the past year might increase the investors reference point against

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<sup>1</sup>It is worth noting that certain results can also be achieved under EUT if the underlying assumptions are more realistic. E.g. Chen *et al.* (2015) recently showed, that the consideration of mortality can explain the preferences for simple guarantees at maturity also in a EUT-framework.

which losses are evaluated. A subsequent drop of in the product's value might then be perceived as a loss, even if the overall performance since the start of the product is still positive. This is related to the concept of mental accounting introduced by Thaler (1985). This concept describes how investors categorize investments in order to monitor the future performance. In a later work, Thaler & Johnson (1990) studied how prior gains and losses affect decision makers and how they frame such problems under Prospect Theory. Arkes *et al.* (2008) provide additional evidence that investors mentally account for previous price changes and therefore regularly adapt their reference point. Benartzi & Thaler (1995) propose the theory of myopic loss aversion, a combination of loss aversion and frequent investment evaluation, and show that an annual evaluation can solve the equity premium puzzle. They argue that mental accounting implies that investors tend to evaluate their investment decision on short evaluation periods and therefore prefer to invest only small fractions of their wealth in risky assets. Benartzi & Thaler (1999) give evidence that investors make less risky choices if they are shown one-year rather than long-term rates of return. Barberis *et al.* (2001) propose a model in terms of asset pricing, in which investors derive utility from annual changes of the value of their financial wealth and Barberis & Xiong (2012) are able to shed light on the disposition effect and other puzzles by introducing the realization utility, which suggest that investors derive utility from interim gains and losses.

If interim changes of the value influence the utility of investors during the investment horizon, it seems only natural that when making the investment decision investors are also affected by potential future interim changes of the value.

Based on these insights, we propose a modification of CPT, which assumes that long-term investors tend to take into account the subjective utility of interim changes of the value

of the contract when making an investment decision. We denote this approach “*Multi Cumulative Prospect Theory*” (MCPT).

As a first application example of MCPT we investigate the demand for different guaranteed products. For the sake of comparability, we apply this approach to the guaranteed contracts presented by Ebert *et al.* (2012), i.e. we consider three different types of guarantees (*roll-up, ratch-up and cliquet*) and a product without guarantee. The roll-up guarantee provides a minimal terminal payoff, which is based on some guaranteed rate of interest. The ratch-up guarantee additionally includes a lock-in feature: The guaranteed benefit is the higher of a fixed guaranteed amount (calculated as in the roll-up case) and the highest investment account value at any pre-specified lock-in date. Finally, the cliquet guarantee credits in each period the higher of a guaranteed rate and the performance of the underlying investment. We analyze these products in a Black Scholes framework without considering mortality or default risk. In this setting we can derive closed-form solutions for the arbitrage free prices of all three products at any valuation date. We use these prices as the basis for the annual changes of the value. We then use Monte Carlo simulations to evaluate these products under EUT, CPT and MCPT. We are able to replicate the results of Ebert *et al.* (2012) under EUT and CPT, particularly the fact that CPT can explain the demand for guarantees but not for the more complex forms of guarantees. However, under our new MCPT approach, the complex products typically dominate the simple products: For investors who evaluate utility considering possible future interim changes, the subjective utility is higher for the complex than for the simple guaranteed products. Also, if only products without guarantee are considered, under MCPT, an equity ratio of 0% is often optimal. This result is in line with the findings of Benartzi & Thaler (1995) and explains the demand for very safe assets even for long term investments that can be observed in many countries. Finally, we present a combined CPT and MCPT approach, where both,

annual changes and the distribution of the terminal wealth are evaluated by the investor. Our analysis under this combined model shows that the demand for complex guaranteed products can be explained, even if the annual price changes only partially influence the total subjective utility.

The remainder of this paper is organized as follows. Section 2 gives a short introduction to CPT. Moreover, we motivate and present our MCPT approach. In Section 3, we apply MCPT to explain the demand for cliquet type guarantees. We specify the considered products and the model for the financial market and present numerical results, as well as sensitivity analyses. Section 4 summarizes and gives an outlook for future research. Finally, the closed-form arbitrage free prices for the considered contracts are given in the Appendix.

## 2 Prospect Theory and Extensions

Prospect Theory (PT) introduced by Kahneman & Tversky (1979) has been developed as one possible way of explaining behavior that can be observed in real life but can not be explained by Expected Utility Theory (EUT). In particular its well-known modification, Cumulative Prospect Theory (CPT) has become very popular.

### 2.1 Cumulative Prospect Theory

Cumulative Prospect Theory is based on the idea, that the subjective utility of an investment  $A$  with final outcomes given by a random variable  $E$  is described by a S-shaped PT value function<sup>2</sup>  $v$  for the gains and losses  $X$  corresponding to the outcomes with respect

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<sup>2</sup>A continuous function  $v : \mathbb{R} \rightarrow \mathcal{I}$  ( $\mathcal{I} \subset \mathbb{R}$  an interval containing 0) is called PT value function, if  $v$  is strictly monotonically increasing,  $v(0) = 0$  (reference-point),  $v(x)$  is strictly convex for  $x < 0$  (decreasing loss sensitivity),  $v(x)$  is strictly concave for  $x > 0$  (decreasing gain sensitivity) and  $|v(-x)| > v(x)$  (loss

to a given reference point  $\chi$ . The gains and losses are described by the random variable  $X := E - \chi$ , and modified by a probability weighting function<sup>3</sup>  $w$ , that overweights (particularly extreme) events with small probabilities and underweights events with high probabilities. A natural and the most prominent choice for an investor's reference point is the initial price of the investment, cf. Kahneman & Tversky (1979), which is also called a Status Quo reference point (SQ), i.e.  $\chi = A_0$  and hence  $X = E - A_0$ , where  $A_0$  denotes the fair value of the investment  $A$  at  $t = 0$ . Now let  $\mu_X$  be the probability measure given by the random variable  $X$ . Then the CPT utility is defined as

$$CPT(X) := \int_{-\infty}^0 v(x)d(w(F(x))) + \int_0^{\infty} v(x)d(-w(1 - F(x))), \quad (1)$$

with  $F(s) = \mathbb{P}(X \leq s) = \int_{-\infty}^s d\mu_X$ . This is a natural generalization (cf. Hens & Rieger (2010)) of the discrete case introduced by Tversky & Kahneman (1992).

## 2.2 Multi Cumulative Prospect Theory

Especially for long term investments, e.g. retirement savings, studies show that investors regularly evaluate their investment. E.g. Benartzi & Thaler (1995) find that the size of the equity premium is consistent with loss averse investors with annual portfolio evaluation (myopic loss aversion) causing even long term investors to choose their strategies based on short evaluation periods. Arkes *et al.* (2008) provide evidence that investors adjust their reference point over the investment horizon. Also, a long term investor usually receives an annual report with current information about the investment. Bellemare *et al.* (2005) find evidence that such interim information alone affects perceived utility (ex post). Mental accounting implies that investors tend to take into account the potential future fluctuation

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aversion).

<sup>3</sup>A continuous function  $w : [0, 1] \rightarrow [0, 1]$  is called probability weighting function, if  $w$  is strictly monotonically increasing,  $w(0) = 0$  and  $w(1) = 1$  and  $w(p) > p$  for  $0 < p \ll 1$  and  $w(p) < p$  for  $0 \ll p < 1$ .

of the contract's value when making an investment decision (ex ante). This motivates that for long term investors, the initial subjective utility of an investment is not only dependent on the distribution of the terminal wealth, but also on the possible future interim changes.

We therefore propose an extension of CPT, which uses CPT utility with multiple reference points and evaluation periods to measure the subjective utility of the potential interim value changes. We refer to this as *Multi Cumulative Prospect Theory (MCPT)*.

We consider an investor and an investment  $A$  with time horizon  $[0, T]$ ,  $T \in \mathbb{N}$ , at time  $t_0 = 0$ . Moreover, to simplify notation, we assume future interim evaluations take place annually. Therefore, we have to introduce a measure for the future annual changes of the value of the investment  $A$ . Since in many countries for fund-linked products the market value of the product has to be communicated to the client on a regular basis, we consider for all  $t \in \{1, \dots, T\}$  the annual gain or loss  $X_t := A_t - \chi_t$ , where  $A_t$  is the fair value of the investment  $A$  at time  $t$  and  $\chi_t$  is the reference point for time  $t$ . In this setting, the natural SQ reference point choice for each period is given by  $\chi_t = A_{t-1}$ . Hence  $X_t = A_t - A_{t-1}$  represents the annual value change with respect to the SQ. Note that this setting implies that investors use different reference points for different points in time. Based on equation (1) we can evaluate the CPT utility at  $t_0 = 0$  of each annual value change  $X_t$  by

$$CPT(X_t) = \int_{-\infty}^0 v(x)d(w(F_t(x))) + \int_0^{\infty} v(x)d(-w(1 - F_t(x))),$$

where  $F_t(x) = \mathbb{P}(X_t \leq x)$  and  $v$  is the investor's value-function.

The MCPT utility at time  $t_0 = 0$  of an investor with investment  $A$  is then given by

$$MCPT(A) := \sum_{t=1}^T \rho^t CPT(X_t) \quad (2)$$

with a discounting parameter  $\rho \in \mathbb{R}_+$ .

## 2.3 Discussion and Choice of the Functions

### 2.3.1 MCPT Preferences

Consider two investments  $A$  and  $B$  with the same time horizon  $[0, T]$ ,  $T \in \mathbb{N}$ . We assume an investor who considers the same future interim evaluation periods (e.g. annually) for both investments. Moreover, the investor makes the investment decision at  $t = 0$  under the assumption that contract will be held until maturity.<sup>4</sup> Recall that in contrast to dynamic choice models, where typically interim evaluation and decision making go hand in hand, the MCPT evaluation periods are not connected with decisions.<sup>5</sup> An MCPT investor with a given value and probability weighting function prefers  $A$  over  $B$  at time  $t_0 = 0$  if  $MCPT(A) > MCPT(B)$ .

A desired and natural consequence of the MCPT definition is that stochastic dominance in the traditional sense is violated i.e. if the terminal value of investment  $A$  stochastically dominates<sup>6</sup> investment  $B$ , investment  $A$  does not necessarily have a higher MCPT utility

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<sup>4</sup>Many long-term investors spend a lot of time before they make a decision for a certain product. But then they stay with it, i.e. the investor does not question the contract after making the decision. Possible explanations are long-term investors are not willing to regularly spend much time on comparing different investment contracts, and that rather high surrender fees in the “old” contract and new commission payments in the new contract make a change less attractive, etc.

<sup>5</sup>Note that allowing for interim decisions would require a consideration of path-dependent decision rules and a detailed execution of dynamic consistency issues leading to further restrictive assumptions or model adjustments. A discussion of dynamic choices for nonexpected utility models is done by Sarin & Wakker (1998) and with focus on changing reference points by Barkan & Busemeyer (1999) and Barkan & Busemeyer (2003).

<sup>6</sup>An investment  $A$  is stochastically dominant over an investment  $B$ , if for every value  $x$ , the probability

than investment  $B$ . This results from MCTP utility being based on the distributions of all annual changes rather than on the distribution of the final outcome only. Nevertheless, since CPT fulfills the stochastic dominance (cf. Levy (2006)) we can conclude that if the annual changes  $X_t^A$  stochastically dominate  $X_t^B$  for all  $t \in \{1, \dots, T\}$ , then  $MCPT(A) > MCPT(B)$ .

### 2.3.2 Reference Point

The choice of the reference point is very important when applying PT or CPT. There are several studies which provide evidence that the SQ plays an outstanding role, cf. Shefrin & Statman (1985) or Spranca *et al.* (1991). Other reasonable static reference points are given by static guaranteed amounts, which describe a minimum requirement, the payoff of a risk free investment or some other static comparison values which indicate e.g. the investor's goal, cf. Heath *et al.* (1999), or an expectation about future outcomes, cf. e.g. Kőszegi & Rabin (2006). Non static variants include the idea that investors adjust their reference point over the investment horizon depending on the evolution, cf. Arkes *et al.* (2008) or Khuman *et al.* (2012), which includes the idea that investors want to retain past gains, or other path depended outcomes of some benchmark. Other studies suggest the use of multiple reference points, cf. Koop & Johnson (2012) or Wang & Johnson (2012), which include a minimum requirement, SQ, and the investor's goal captured by a double S-shaped value function. Moreover, studies suggest that investors can simultaneously consider multiple reference points without combining them, cf. e.g. Ordóñez *et al.* (2000). Knoller (2016) shows that adding a goal that serves as cushion can partially explain the high demand for guarantees in annuity products.

With increasing regulation, more and more countries require that investors receive regular

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to obtain more than  $x$  is larger or equal for  $A$  than for  $B$  and there exists at least some value  $x$ , such that this probability is strictly larger. Cf. Hens & Rieger (2010).

information about the value of their contract. Therefore, one might expect that moving and multiple reference points become more important particularly for long term contracts e.g. in old age provision or retirement planning. MCPT takes this into account by using different reference points for different future points in time. But in contrast to a reference point adaptation or the use of multiple reference points only, MCPT investors anticipate already in the investment decision their future annual evaluations based on potential future reference points and outcomes.

### 2.3.3 Value and Probability Weighting Function

There is a variety of literature on the choice of the value function in PT, e.g. Stott (2006). For the purpose of this paper we focus on the most common PT value function in finance, the power value function, which is defined as

$$v(x) := \begin{cases} x^a, & x \geq 0 \\ -\lambda|x|^b, & x < 0 \end{cases}, \quad (3)$$

where  $\lambda > 0$  is the loss aversion parameter (in PT typically  $\lambda \approx 2$ ) and  $a \in \mathbb{R}_+$  and  $b \in \mathbb{R}_+$  effect the different sensitivity to losses and gains. In PT, typically  $a, b \leq 1$  and it is very common to set  $a = b$ , such that  $\lambda$  becomes the only parameter that affects the difference between the treatment of gains and losses. This assumption is based on several experimental and empirical results e.g. in Tversky & Kahneman (1992), Camerer & Ho (1994) or Tversky & Fox (1995).

As probability weighting function we use the Tversky Kahneman version:

$$w(p) := \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \text{ with } \gamma \in (0.28, 1], \quad (4)$$

where the lower boundary for  $\gamma$  is chosen, such that  $w(p)$  is strictly monotonically increasing for  $p \in [0, 1]$ . Similar as for the value function we refrain from a different treatment of gains and losses with respect to the probability weighting. Note that  $\gamma = 1$  represents the case without probability weighting.

## 2.4 Combining CPT and MCPT

The MCPT utility reflects the subjective utility created by potential interim changes. Nevertheless, the terminal value plays an outstanding role. Therefore, we propose a combination of CPT and MCPT, where investors consider both, interim value changes  $X_t = A_t - A_{t-1}$  and the terminal value change  $X = A_T - A_0$ . We define this combination by

$$CPT^{com}(A) := sMCPT(A) + (1 - s)CPT(X) \quad (5)$$

with  $s \in [0, 1]$  controlling the influence of the interim value changes on the total subjective utility.

## 3 Application of MCPT: Explaining the Demand for Cliquet-Type Guarantees

In this Section we apply MCPT to three guaranteed products (*roll-up*, *ratch-up* and *cliquet*) and a product without guarantee. If not stated otherwise, we follow Ebert *et al.* (2012) in this section.

### 3.1 Financial Market

We assume a Black Scholes financial market model (Black & Scholes (1973)). We consider a filtered probability space  $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$  with a finite time horizon  $T \in (0, \infty)$  and a real world measure  $\mathbb{P}$  satisfying the usual assumptions.  $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}$  and the  $\sigma$ -algebra  $\mathcal{F}_t$  contains the available information at time  $t$ . The risky asset  $S$  is given by

$$dS_t = S_t(\mu dt + \sigma dW_t), \quad S_0 = 1,$$

where  $W$  is a standard Brownian motion with respect to  $\mathbb{P}$ . The risk free asset  $B$  is given by  $dB_t = B_t r dt$ . We assume  $\mu > r \geq 0$  and  $\sigma > 0$ . Moreover, we define  $\theta \in [0, 1]$  to be the fraction of wealth invested in the risky asset  $S$ , and  $1 - \theta$  the fraction invested in the risk free asset  $B$ . We assume continuous rebalancing to keep these ratios stable. The portfolio value process  $V$  is then given by the  $\mathbb{P}$ -dynamic

$$dV_t(\theta) = V_t(\theta) \left( \theta \frac{dS_t}{S_t} + (1 - \theta) \frac{dB_t}{B_t} \right), \quad V_0(\theta) = 1,$$

which has the solution:  $V_t(\theta) = V_0(\theta) e^{(r + \theta(\mu - r) - \frac{1}{2}\theta^2\sigma^2)t + \theta\sigma W_t}$ .

### 3.2 Contract Types

We study four investment (or insurance) contracts with different guarantee features and benefits at a fixed future (retirement) date  $0 < T < \infty$  and inception  $t_0 = 0$ . The premium  $P$  paid at  $t_0 = 0$  is assumed to be 1. The investment premium  $\alpha$  describes the fraction of the premium allocated to the investment account  $V$ , while the remaining part  $1 - \alpha$  is used to finance the guarantee. Moreover, we define lock-in dates  $t_1, \dots, t_n = T$  as the endpoints of  $n$  equidistant subintervals of  $[0, T]$  with length  $\Delta t = \frac{T}{n}$ .

The first considered guarantee feature is the *roll-up*. Its payoff at maturity  $T$  is given by

$$A_T^{rol} := \max(e^{gT}, \alpha V_T) = \alpha V_T + [e^{gT} - \alpha V_T]^+.$$

The roll-up essentially provides a guaranteed rate  $g$  on the original premium. It is a frequently offered guarantee feature, e.g. in the context of variable annuities (cf. e.g. Bauer *et al.* (2008)).

Estep & Kritzman (1988) argue that investors are not only interested in the protection of a pre-specified fixed level, but also in the protection of interim gains. Guarantees like ratchet or cliquet features are able to incorporate this effect. Therefore, as a second product, we consider the so-called *ratch-up*, which is a combination of a roll-up and a ratchet feature with the following payoff:

$$A_T^{rat} := \max(e^{gT}, \alpha V_{t_1}, \dots, \alpha V_T) = \alpha V_T + [\max(e^{gT}, \alpha V_{t_1}, \dots, \alpha V_{t_{n-1}}) - \alpha V_T]^+$$

This product essentially pays the highest portfolio value at any lock-in date or a roll-up with rate  $g$ , whichever is higher.

The third and final guaranteed product will be referred to as *cliquet* product with payoff:

$$A_T^{cli} := \prod_{i=1}^n \max\left(e^{g\Delta t}, \alpha^{\frac{1}{n}} \frac{V_{t_i}}{V_{t_{i-1}}}\right) = \prod_{i=1}^n \left(\alpha^{\frac{1}{n}} \frac{V_{t_i}}{V_{t_{i-1}}} + \left[e^{g\Delta t} - \alpha^{\frac{1}{n}} \frac{V_{t_i}}{V_{t_{i-1}}}\right]^+\right)$$

In each period, this product locks in the higher of a guaranteed rate  $g$  and the performance of the underlying portfolio  $V$  (the latter only with respect to a portion  $\alpha^{\frac{1}{n}}$  of the investment premium). This representation of a cliquet product from Ebert *et al.* (2012) is rather unusual. However, for  $\tilde{g} = g - \log(\alpha)$ , this representation coincides with the more common

representation

$$A_T^{\widetilde{cli}} := \alpha \prod_{i=1}^n \max \left( e^{\widetilde{g}\Delta t}, \frac{V_{t_i}}{V_{t_{i-1}}} \right) = \alpha \prod_{i=1}^n \left( \frac{V_{t_i}}{V_{t_{i-1}}} + \left[ e^{\widetilde{g}\Delta t} - \frac{V_{t_i}}{V_{t_{i-1}}} \right]^+ \right).$$

In the more common representation, the contract in each period simply earns the greater of the guaranteed rate and the performance of the underlying portfolio.

Besides the contracts with guarantee feature, we consider a contract without guarantee investing in the underlying  $V$ , which we refer to as *constant mix (cm)* contract. Note that here  $\alpha = P = 1$  and therefore obviously  $A_T^{cm} = V_T$ .

We will only consider fair contracts with an identical initial arbitrage free price of 1, i.e. we consider contracts  $c \in \{rol, rat, cli\}$  with  $A_0^c(g, \alpha, \theta) = 1$ , where  $(g, \alpha, \theta) \in [-\infty, r] \times (0, 1] \times [0, 1]$ . Closed form solutions for the arbitrage free prices of the different products at  $t = 0$  and at each lock-in date  $t_1, \dots, t_{n-1}$  are given in Appendix A. In the analysis, we use the approach of Ebert *et al.* (2012), i.e. we fix  $(\alpha, \theta)$  and determine for each contract the value of  $g \in [-\infty, r]$ , that makes the contract fair.<sup>7</sup> This represents a more intuitive choice in a behavioral context than fixing  $g$ , since  $1 - \alpha$  denotes the fair value of each guarantee and  $\theta$  is a measure of the upside potential of the underlying.

### 3.3 Results

In this Section, we present numerical results for the following financial market parameters:  $\mu = 0.06$ ,  $\sigma = 0.3$ ,  $r = 0.03$  and  $T = n = 5$ , i.e. the lock-in dates are at  $t = 1, \dots, 5$  and the periods are one year. We will perform sensitivity analyses in Section 3.4 including longer

<sup>7</sup>If such a  $g$  exists, then it is unique. Moreover, within this setting for the roll-up and the cliquet features such a  $g$  exists for all combinations of  $\alpha$  and  $\theta$ . But for large  $\theta$  and small  $\alpha$  no solution exists for the ratch-up feature.

time horizons. For the Monte Carlo sample size we use  $l = 20,000$  and the lower bound for  $\alpha$  is chosen to be 0.6.

### 3.3.1 Guarantee Levels and Terminal Distributions

Before analyzing the utility of the different contracts and the question, which guarantee type is preferred by which investor, we take a closer look at the features of the different contracts to point out the differences and similarities.

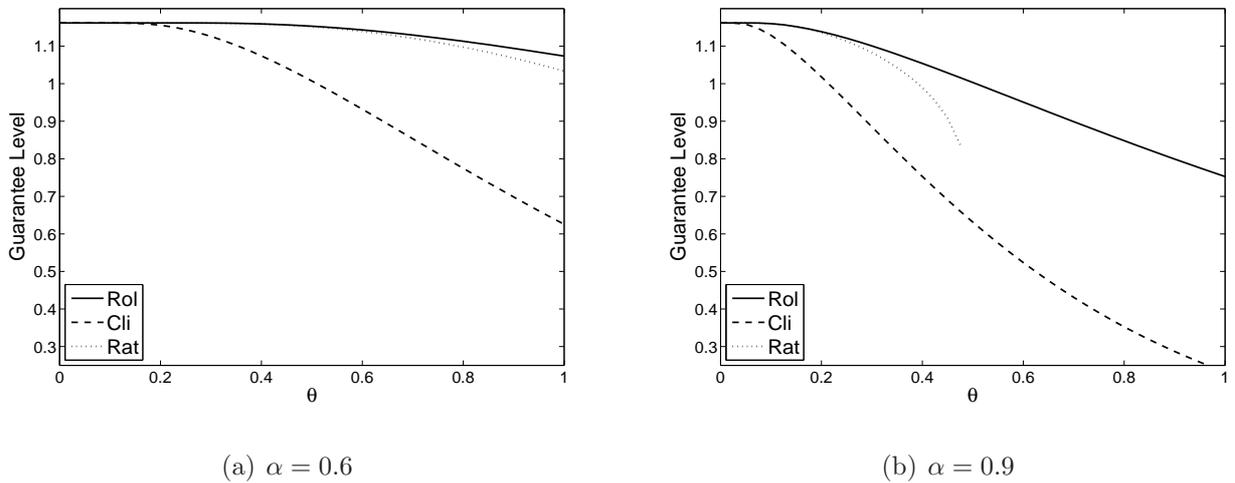
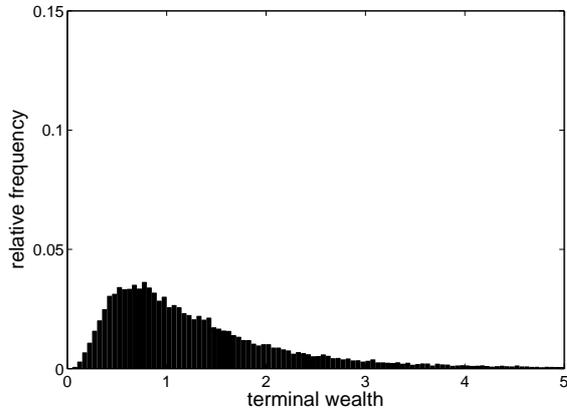
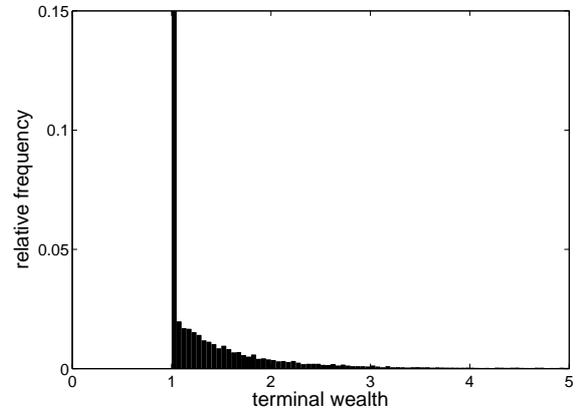
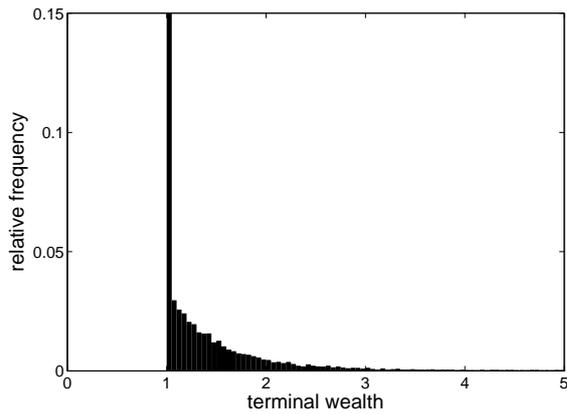
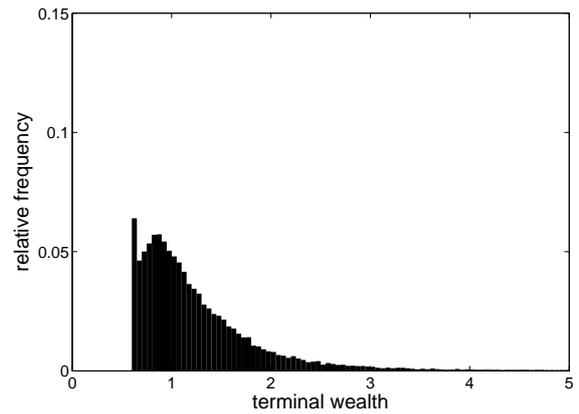


Figure 1: Fair guarantee levels, i.e.  $e^{gT}$ , for  $\alpha = 0.6$  (left panel) and  $\alpha = 0.9$  (right panel) as a function of  $\theta$ .

Figure 1 displays the different guarantee levels ( $e^{gT}$ ) of the contracts for different fractions  $\theta$  invested in the risky asset. When compared to the roll-up, the ratch-up also locks in past peaks, which is more expensive. The cliquet guarantee is even more expensive since each period's performance is maximized with the guaranteed return. Therefore for given  $(\alpha, \theta)$  we have  $g_{rol} \geq g_{rat} \geq g_{cli}$ . Moreover, for very small values of  $\theta$ , the guarantee level increases slightly in  $\theta$ . This effect, which results from diversification, is stronger for longer time horizons. After reaching a peak, the guarantee level decreases for increasing  $\theta$ .



(a) Constant Mix (Pure Stock)

(b) Roll-up with  $g_{fair} = 0.0142$ , guarantee level 1.0735 and guarantee level frequency 77.29%.(c) Ratch-up with  $g_{fair} = 0.0066$ , guarantee level 1.0337 and guarantee level frequency 66.77%.(d) Cliquet with  $g_{fair} = -0.0938$ , guarantee level 0.6257 and guarantee level frequency 2.78%.Figure 2: Payoff distribution of the different contracts for  $\alpha = 0.6$  and  $\theta = 1$ .

To get an impression of the risk-return-characteristics of the terminal payoffs of the different contracts, Figure 2 displays the payoff distributions for  $\alpha = 0.6$  and  $\theta = 1$ . The upper left panel shows the payoff distribution of the pure stock investment, while the other three panels display the different guaranteed products.

The guarantee levels are reflected in the terminal payoff distributions. All distributions are right-skewed. The roll-up contract has the highest guarantee level (1.0735), and also the highest guarantee level frequency (77.29%), which is the probability that the terminal value coincides with the guarantee. It is followed by the ratch-up contract (1.0337 resp. 66.77%). The lower guarantee frequency for the ratch-up contract is a consequence of its design, which locks in past peaks, if they exceed the guarantee level. As a consequence of the lower guarantee level, the right tail of the ratch-up is heavier than the right tail of the roll-up, which indicates higher upside potential. The guarantee level of the cliquet contract amounts to 0.6257 and is by far the lowest. In turn, the guarantee level frequency is very low (2.78%). The cliquet contract protects only against rather high losses, but does so in each year, whereas the other two guaranteed contracts guarantee even a small gain compared to the initial premium. For smaller values of  $\theta$ , the guarantee level of the cliquet is higher, inducing also a higher guarantee level frequency, e.g. for  $\theta = 0.5$  the guarantee level of the cliquet contract is 1.0014 and the guarantee level frequency is 14.27%. Note that the guarantee level frequency is significantly lower for the cliquet contract, even for similar guarantee levels. This due to the fact, that for the cliquet contract one period with a good performance is sufficient for a terminal value exceeding the guarantee.

### 3.3.2 Annual Price Changes

Since MCPT utility is driven by annual price changes, we first illustrate the differences between the contracts in this respect. Percentiles of the distribution of the annual price changes  $X_t = A_t^c - A_{t-1}^c$  with  $t \in \{1, \dots, T\}$  and  $c \in \{cm, rol, rat, cli\}$  are displayed exemplarily for the year  $t = 3$  in Figure 3 for different choices of  $\alpha$  and  $\theta$ . In the case  $\alpha = 0.6$  and  $\theta = 0.5$  (upper left panel), the guarantee levels are rather similar: 1.1460 for the roll-up, 1.1431 for the ratch-up and 0.9673 for the cliquet contract. The Figure shows that the annual price changes for the guaranteed contracts are subject to less fluctuations

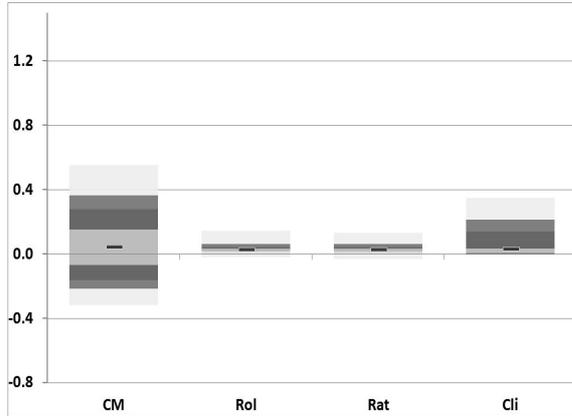
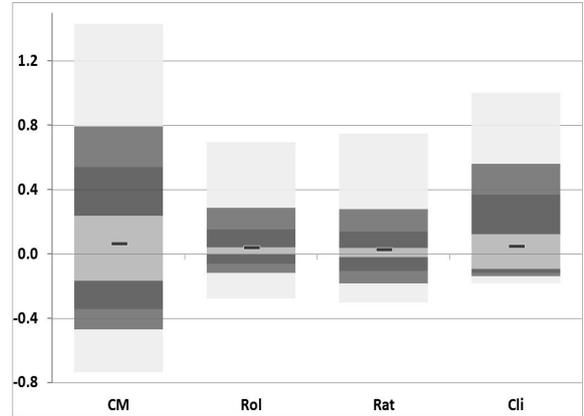
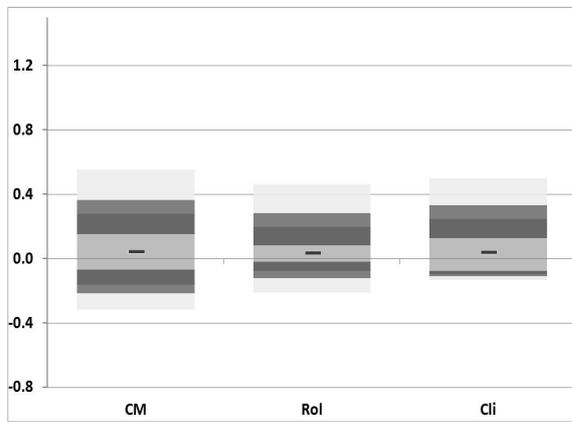
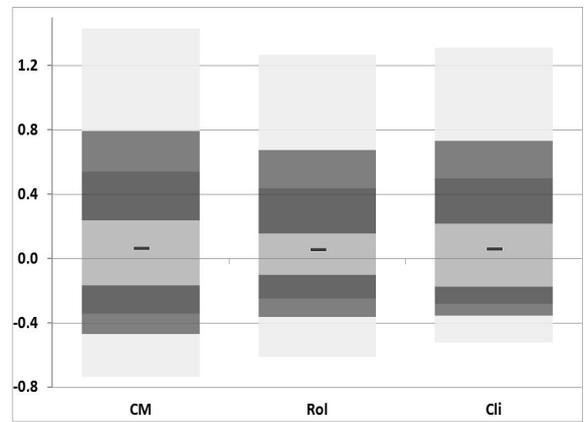
(a)  $\alpha = 0.6, \theta = 0.5$ (b)  $\alpha = 0.6, \theta = 1$ (c)  $\alpha = 0.9, \theta = 0.5$ (d)  $\alpha = 0.9, \theta = 1$ 

Figure 3: Percentiles of the distribution of annual changes in the 3rd year ( $X_3$ ) of the fair prices of the different contracts for different parameters  $\alpha$  and  $\theta$ . The bars indicate the 1% – 5%, 5% – 10%, 10% – 25%, 25% – 75%, 75% – 90%, 90% – 95% and 95% – 99% percentiles and the black lines indicate the mean.

than for the contract without guarantee (constant mix). While in the case  $\alpha = 0.6$  and  $\theta = 0.5$  negative price changes are very unlikely, the upper right panel shows that even for this value of  $\alpha$  the probability of negative price changes increases if  $\theta$  is increased.

The cliquet contract shows a different structure in the annual changes than the other two guaranteed contracts. It significantly reduces the probability for strong negative price

changes, while the probability of medium negative price changes and the potential for large positive price changes is higher than with the other two guaranteed contracts, i.e. the distribution of the annual price changes is more right-skewed. This is an important difference, since CPT investors tend to prefer right skewed distributions (cf. Barberis & Huang (2008) and Ebert & Strack (2015)). In the case  $\alpha = 0.9$  (lower panels), there exists no solution for the ratch-up contract for both,  $\theta = 0.5$  and  $\theta = 1$ . The distributions of the annual price changes of the roll-up and the cliquet contract spread more widely than for the previous cases due to the higher investment into the underlying investment account. Also, the distribution of the annual price changes of the cliquet contract is not as right-skewed as in the previous cases because of the significantly lower guarantee level.

### 3.3.3 Expected Utility Theory Analysis

Ebert *et al.* (2012) show that for a CRRA EUT investor with reasonable risk aversion parameter, all guaranteed contracts create disutility when compared to a constant mix contract. This is also consistent to the fact that for a CRRA EUT investor any deviation from the optimal Merton strategy leads to disutility (cf. also Merton (1971) or Tepla (2001)). These findings can be replicated in our model and hold for all pairs  $(\alpha, \theta)$  and all reasonable financial market parameters.

### 3.3.4 Cumulative Prospect Theory Analysis

We were able to replicate the CPT findings from Ebert *et al.* (2012). Additionally, we performed analyses in a model with reference point adaptation. The results that we have replicated are: A CPT investor prefers either the constant mix or the roll-up contract, i.e. if a guaranteed contract is preferred over the constant mix contract, then the roll-up always dominates the other guarantees even if each contract is specified with its optimal (i.e. CPT utility maximizing) guarantee level. In particular for high values of  $\theta$  the roll-

up outperforms the constant mix contract. Furthermore, the results show that in these cases either a guarantee level equal to the reference point or an insurance against large losses only is optimal for the CPT investor. The main results did not change under different static reference point choices, different CPT parameters or financial market parameters.

In addition, we have considered a model with reference point adaptation as proposed by Khuman *et al.* (2012), i.e.  $\chi = s \max(A_0^c, \dots, A_T^c) + (1 - s)A_0^c$  and  $s \in [0, 1]$  for  $c \in \{cm, rol, rat, cli\}$ . The first part of the reference point includes the idea of retaining past peaks, whereas the second part is again the SQ. Our simulations under this model indicate, that this reference point adaptation is also not able to explain the demand for the more complex guaranteed products. Although the more complex guarantees do better than the roll-up contract for most of the combinations, for these combinations the constant mix contract outperforms all guaranteed contracts.

### 3.3.5 Multi Cumulative Prospect Theory Analysis

This Section presents the main results of our paper. We use MCPT as described in Section 2.2 to analyze the influence of the annual price changes on the subjective utility. We use the same CPT parameters as in the pure CPT case, i.e. we fix  $a = 0.88$ , as suggested by Tversky & Kahneman (1992) and perform analyses for different values of  $\lambda$  and  $\gamma$ . Moreover, we consider the case without discounting, i.e.  $\rho = 1$ . As explained in Section 2.2, for each period we use the annual price change  $X_t = A_t^c - A_{t-1}^c$  based on the SQ reference point for time  $t$  for  $c \in \{cm, rol, rat, cli\}$ . We will derive certainty equivalent contracts ( $CE^M$ ) with a fixed annual return  $r^{CE}$ , i.e.  $r^{CE}$  describes the fix annual return that an investor would regard equally desirable as the considered contract  $c$ . Therefore

$X_t^{CE} = A_{t-1}^{CE}(e^{r^{CE}} - 1)$  and  $\sum_{t=1}^T \rho^t CPT(X_t^{CE}) = MCPT(A^c)$ , which leads to:

$$MCPT(A^c) = \begin{cases} \sum_{t=1}^T \rho^t \left( X_1^{CE} e^{r^{CE}(t-1)} \right)^a, & MCPT(A^c) \geq 0 \\ -\lambda \sum_{t=1}^T \rho^t \left| X_1^{CE} e^{r^{CE}(t-1)} \right|^a, & MCPT(A^c) < 0 \end{cases}$$

We solve the equation numerically for each contract to obtain the corresponding fix annual return  $r^{CE}$ .

Figure 4 illustrates the influence of the different CPT features on the MCPT value for  $\alpha = 0.9$ . The upper left panel shows the MCPT without loss aversion and probability weighting, such that the only considered CPT feature is the S-shaped value function. Similar to the EUT and CPT case, for all contracts the certainty equivalent return increases with increasing  $\theta$ , the optimal value of  $\theta$  is 1 and the constant mix strategy dominates the other contracts.

The upper right panel additionally includes loss aversion with  $\lambda = 2.25$ . In this setting, the constant mix contract is dominated by the guaranteed contracts, because they reduce the probability of negative annual price changes (cf. Section 3.3.2). This is also the reason why the roll-up and the ratch-up dominate the cliquet contract in this case. Especially for higher values of  $\theta$ , the probability of annual losses is higher for the cliquet contract than for the other two guaranteed contracts, since (as explained in Section 3.3.2) the cliquet product has a significantly lower minimum guarantee level and hence a higher potential for a price drop. As for the CPT case, the certainty equivalent return increases slightly for low values of  $\theta$  and decreases for high values of  $\theta$ , where the value of  $\theta$  at the peak is higher, if the guarantee level is higher.

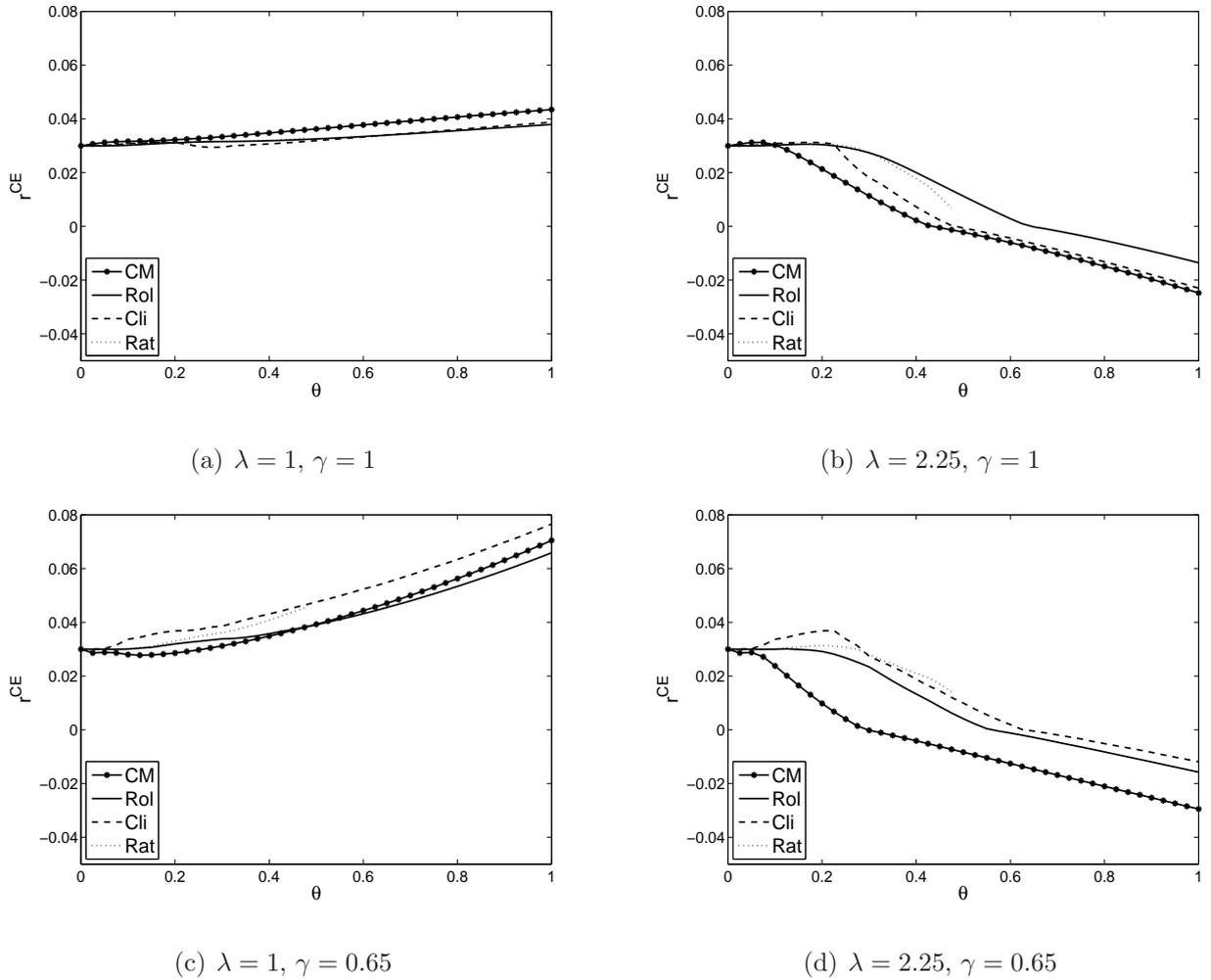


Figure 4:  $r^{\text{CE}}$  for the constant mix and the three guaranteed products as a function of  $\theta$  for  $\alpha = 0.9$ .

The lower left panel shows the influence of the probability weighting without loss aversion, i.e.  $\gamma = 0.65$  and  $\lambda = 1$ . Here, the cliquet contract dominates the other contracts for all values of  $\theta$ . This can again be explained by the annual price change distributions (cf. Section 3.3.2) and the fact that CPT particularly overweights extreme events that occur with low probability. The annual price changes of the cliquet contract are extremely right-skewed, i.e. they include some relatively high gains with low probabilities. These outcomes get overweighted and generate a higher certainty equivalent return. The constant mix con-

tract also contains high gains, that happen with low probabilities, but also high losses, that happen with low probabilities, which reduce the certainty equivalent return. The certainty equivalent return of all contracts increases with increasing  $\theta$  and the optimal value of  $\theta$  is 1.

Finally, the lower right panel includes all CPT features. Here, the MCPT certainty equivalent return of the more complex guaranteed products (ratch-up and cliquet) exceeds the roll-up and the constant mix contract. Due to the loss aversion, the constant mix performs worst. The overall optimal certainty equivalent return is reached by the cliquet contract, which also performs best for most values of  $\theta$ . However, the optimal value of  $\theta$ , i.e. the fraction invested in the risky asset, is lower than in the CPT case. This is due to the fact, that CPT investors with a longer evaluation period prefer more risky assets (cf. Benartzi & Thaler (1995) or Berkelaar *et al.* (2004)) than in our setting with an annual evaluation.

Figure 5 illustrates the same effects for  $\alpha = 0.6$ , i.e. for higher guarantee levels. It is worth noting that especially for high values of  $\theta$ , the guaranteed contracts perform much better, than in the case  $\alpha = 0.9$ . The upper right plot shows that the higher guarantee levels reduce the losses in the annual price changes, that caused disutility in the  $\alpha = 0.9$  case. Therefore the MCPT certainty equivalent return difference between the constant mix and the guaranteed contracts is even larger in this case.

In a next step we look at different levels of  $\alpha$  and  $\theta$  simultaneously. Figure 6 displays the values of the MCPT certainty equivalent return for the four different contracts as a function of  $\alpha$  and  $\theta$ . Note, that the lower bound for  $\alpha$  is chosen to be 0.6.

For each fixed level of  $\alpha$ , the cliquet contract generates the highest certainty equivalent return. The overall maximum certainty equivalent return for the cliquet and therefore for

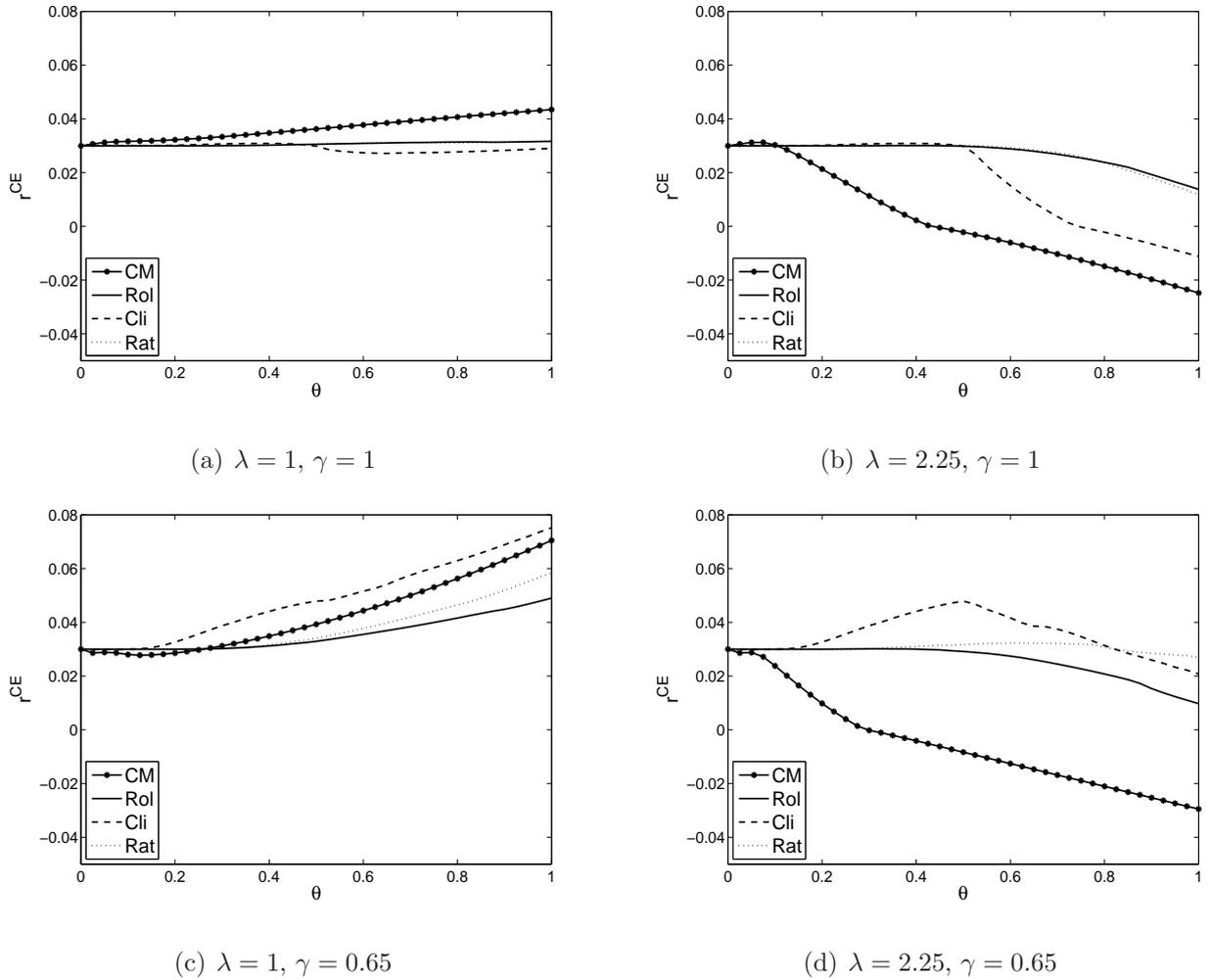
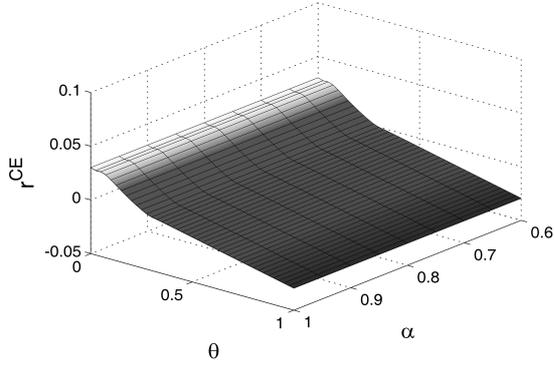


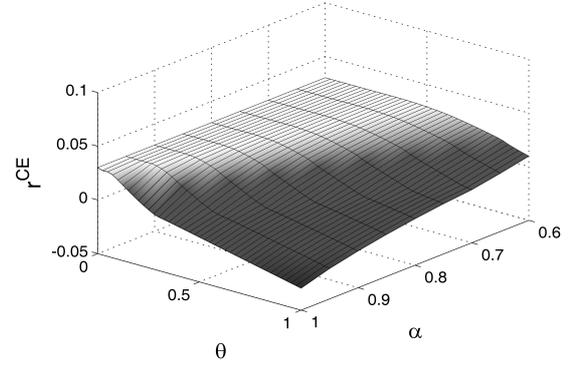
Figure 5:  $r^{CE}$  values for the constant mix and the three guaranteed products as a function of  $\theta$  for  $\alpha = 0.6$ .

all contracts is 4.79% for  $\alpha = 0.6$  and  $\theta \approx 0.5$ .<sup>8</sup> Moreover, the cliquet performs better than both, the roll-up and the constant mix contract for almost all pairs  $(\alpha, \theta)$ . The maximum for the roll-up contract is 3.01% for  $\alpha \approx 0.75$  and  $\theta \approx 0.325$  and as seen before the maximum for the constant mix contract is 3% at  $\theta = 0$ . This means that for an investor, who takes annual changes into account, the risk free asset is the most attractive

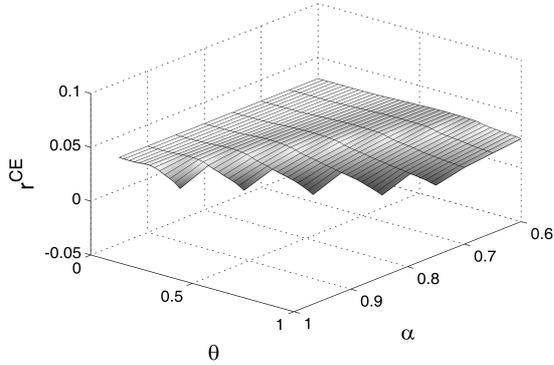
<sup>8</sup>If we allow for all levels of  $\alpha$ , this remains true and the optimal value of the cliquet contract is at  $\alpha \approx 0.2$  and  $\theta = 1$ .



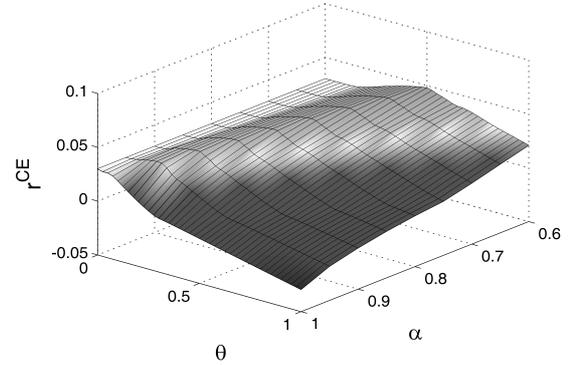
(a) Constant Mix



(b) Roll-up



(c) Ratch-up



(d) Cliquet

Figure 6:  $r^{CE}$  values for the constant mix and the three guaranteed products as a function of  $\theta$  and  $\alpha$ . CPT parameters:  $\lambda = 2.25$ ,  $\gamma = 0.65$ .

underlying for a contract without guarantee. This finding might explain the demand for very safe assets, that can be observed in many markets.

### 3.3.6 A Combined CPT and MCPT Analysis

As described in Section 2.4, MCPT reflects the utility created by annual price changes. Investors might consider both, annual price changes and terminal value. Therefore, we now analyze combinations of the CPT and the MCPT utility, i.e. we look at  $CPT^{com}(A^c) :=$

$sMCPT(A^c) + (1 - s)CPT(X)$  with  $s \in [0, 1]$  and  $X = A_T^c - \chi$ .<sup>9</sup> Again, we will derive certainty equivalent contracts ( $CE^{com}$ ) with a fixed annual return  $r^{CE}$  that is determined by:

$$CPT^{com}(A^c) = \begin{cases} s \sum_{t=1}^T \rho^t \left( X_1^{CE} e^{r^{CE}(t-1)} \right)^a + (1 - s) (X^{CE})^a, & CPT^{com}(A^c) \geq 0 \\ -\lambda \left( s \sum_{t=1}^T \rho^t \left| X_1^{CE} e^{r^{CE}(t-1)} \right|^a + (1 - s) |X^{CE}|^a \right), & CPT^{com}(A^c) < 0. \end{cases}$$

The upper panels in Figure 7 illustrate the certainty equivalents in the combined model for the case  $\alpha = 0.6$  and lower panels for the case  $\alpha = 0.9$ , with  $s = 0.3$  and  $s = 0.5$ . The lower panels in Figure 7 show for the case  $\alpha = 0.9$ , that both, the cliquet and the ratch-up contract outperform the constant mix contract for all values of  $\theta$  in both considered combinations. Moreover, the ratch-up for  $s = 0.3$  resp. the cliquet contract for  $s = 0.5$  generate the highest combined certainty equivalent return. Similar results can be seen in the upper panels for the case  $\alpha = 0.6$ . In this case the constant mix and even the roll-up are outperformed for all values of  $\theta$  by at least one of the more complex guaranteed contracts. The highest combined certainty equivalent return in this case is given by the ratch-up with  $\theta = 1$  in both considered combinations. Moreover, we have calculated the certainty equivalent return as a function of  $\theta$  and  $\alpha$  similar to the MCPT case for  $s = 0.3$  and  $s = 0.5$ . The results show that the described findings also hold for other parameter combinations and the highest overall combined certainty equivalent return is generated by the ratch-up for  $s = 0.3$  ( $r^{CE} = 4.85\%$  for  $\theta = 1$  and  $\alpha = 0.6$ ) resp. the cliquet contract for  $s = 0.5$  ( $r^{CE} = 4.27\%$  for  $\theta = 0.5$  and  $\alpha = 0.6$ ).

We can conclude that in contrast to EUT and CPT (with and without reference point

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<sup>9</sup>Note that in this setting  $\chi = \chi_1 = 1$ , where  $\chi_1$  represents the reference point in MCPT at time 1.

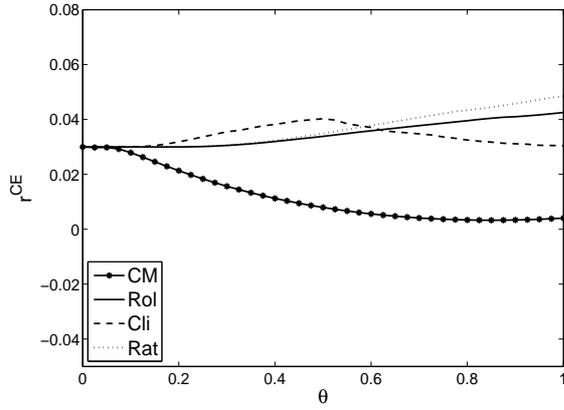
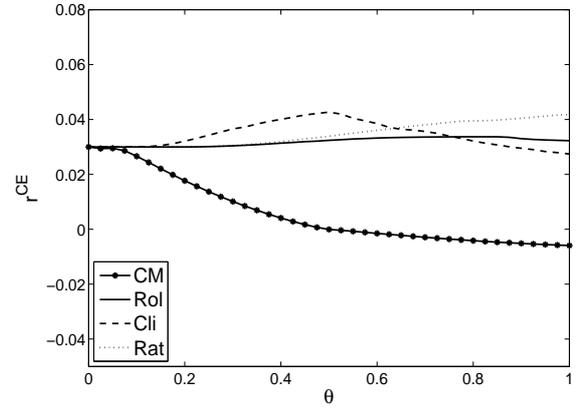
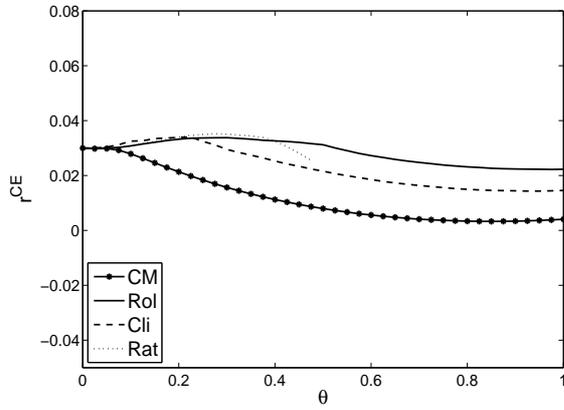
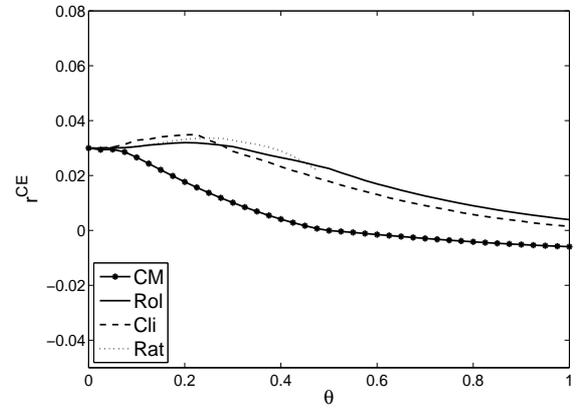
(a)  $\alpha = 0.6$ :  $r^{CE}$  with  $s = 0.3$ (b)  $\alpha = 0.6$ :  $r^{CE}$  with  $s = 0.5$ (c)  $\alpha = 0.9$ :  $r^{CE}$  with  $s = 0.3$ (d)  $\alpha = 0.9$ :  $r^{CE}$  with  $s = 0.5$ 

Figure 7:  $r^{CE}$  with  $s = 0.3$  and  $s = 0.5$  for the constant mix and the three guaranteed products as a function of  $\theta$  and  $\alpha = 0.6$  (upper panels) resp.  $\alpha = 0.9$  (lower panels). CPT parameters:  $\lambda = 2.25$ ,  $\gamma = 0.65$ .

adaptation), MCPT can explain the demand for more complex guaranteed contracts (cli-quet and ratch-up). This remains true even if value fluctuations only partly influence the investor's subjective utility

### 3.4 Sensitivity Analysis

We have performed sensitivity analyses with respect to different parameters. First, we used different financial market parameters, i.e.  $\mu$ ,  $\sigma$  and  $r$ . Generally, under reasonable parameter settings our main findings for the MCPT are stable. For a fixed  $\mu$ , we find that an increasing volatility makes the simple products even less attractive, since more low probability events of large losses happen for the constant mix contract and the roll-up includes less low probability annual gains compared to the other two guaranteed contracts. Moreover, the better the market environment, i.e. higher  $\mu$  and lower  $\sigma$ , the higher the optimal fraction  $\theta$  invested in the risky asset, while the differences between the products remain rather stable. Moreover, we have performed simulations with longer investment horizons  $T$  (10 and 20 years). The main findings prevail, i.e. the more complex guaranteed contracts still outperform the simple contracts. E.g. for  $T = 10$  the overall maximum certainty equivalent annual return  $r^{CE}$  is 4.15% generated by the cliquet contract for  $\alpha = 0.6$  and  $\theta \approx 0.33$ . Contrary to the CPT, MCPT investors reduce (if possible) the fraction invested in the risky asset  $\theta$  for longer investment horizons to obtain the maximum MCPT certainty equivalent return for a fixed  $\alpha$ . However, the annualized guarantee rates  $e^g$  are rather similar. E.g. the annualized guarantee rate of the overall optimal cliquet contract in the case of  $T = 5$  is 1.0014 ( $\alpha = 0.6$ ,  $\theta \approx 0.5$ ) and 1.0025 in the case of  $T = 10$  ( $\alpha = 0.6$ ,  $\theta \approx 0.33$ ). Using different empirically reasonable value function parameters  $a \in [0.8, 1]$  (cf. Tversky & Kahneman (1992), Birnbaum & Chavez (1997) or Abdellaoui (2000)) does not change the findings significantly. We also repeated the analysis for different discounting factors  $\rho$ . We have observed that for all reasonable values for the time discounting parameter  $\rho < 1$  the influence is negligible.

Last, we have investigated the influence of the reference point adaptation, i.e. the use of annual price changes as a basis for the annual CPT evaluation. For this, we ran the MCPT

simulations also with reference points fixed to the initial fair price, i.e.  $\chi_1 = \chi_2 = \dots = \chi_T = 1$ . This means that investors still evaluate annually, but their reference point remains equal to the initial fair price of the contract. In this setting, we find that all guaranteed contracts outperform the constant mix contract in all cases. But without reference point adaptation, we also find that the roll-up dominates the more complex guaranteed contracts for all fixed levels of  $\alpha$ . This confirms that the reference point adaptation, in particular with respect to annual changes, is the feature of MCPT, that can explain the demand for more complex guaranteed products.

## 4 Conclusion and Outlook

In this paper, we have proposed an extension of CPT that we call Multi Cumulative Prospect Theory (MCPT). It is based on the CPT utility generated by e.g. annual changes of the contract value. This is motivated by the fact that investors tend to regularly re-evaluate their investment and adapt their reference point based on the evolution of the investment value. We propose that investors who are attracted by specific contract features, like annual lock-in guarantees, might be particularly inclined to take into account the subjective utility of potential future fluctuations of the contract value when making investment decisions. MCPT measures the subjective utility generated by potential interim changes of the contract value. Nevertheless, the terminal value has an outstanding role when making an investment decision. Therefore, we also propose a combination of CPT and MCPT, which considers both, annual price changes and terminal value.

As an application we have analyzed three guaranteed products, which are common in many markets (roll-up, ratch-up and cliquet) and a contract without guarantee (constant mix). First, we could confirm previous results, that neither EUT nor CPT can explain the de-

mand for the more complex types of guarantees. Moreover, we have performed an analysis with a CPT reference point adaptation and have found that this extension is (at least in our setting) also not able to explain the demand for the more complex guaranteed contracts.

When applying MCPT, the more complex guaranteed contracts, in particular the cliquet contract, outperform the other contracts (roll up and constant mix) in all considered cases. This is mainly caused by the more right-skewed distribution of the annual changes in the value of the cliquet contract compared to the other contracts. Hence, our approach is able to explain the demand for these contracts. Moreover, the contract without guarantee creates the more disutility, the higher the fraction invested in the risky asset  $\theta$ . If only products without guarantee are offered, then an MCPT investor prefers an investment in the risk free asset. Therefore, our approach can also explain the very large holdings of safe assets that can be observed in many countries.

Additionally, we have analyzed the contracts under the combined model, which considers both, the terminal value of the investment, and the annual value changes. Our results show that also in this combined model investors may have a preference for the more complex guaranteed products. This means that demand for more complex guarantees exists even if value fluctuations only partly influence the investor's subjective utility.

The analyses in this paper provide a first indication that MCPT has some descriptive power in particular for long-term investments. Still, the influence of value fluctuations on the subjective utility is not completely understood. Future research should therefore address the following questions: For which types of contract features do investors tend to take into account value fluctuations when making the investment decision? How strong is the influence of the value fluctuations on the subjective utility compared to the influence of

the terminal value? Moreover, we have only analyzed the MCPT-utility at time  $t = 0$ . An analysis of the MCPT-utility during the contract duration at  $t > 0$  on the issue of optimal switching between different types of product could give additional insights on client's surrender behavior. Also, we have only considered a restricted set of contracts and compared them through simulations, leaving several theoretical questions like: Under which general conditions does an optimum for a MCPT investor exist and what general properties hold for the MCPT? Furthermore, there are several other aspects that might affect the results and should therefore be considered in future research. Such aspects include the influence of a more dynamic reference point, that depends on the complete history of previous gains and losses (as opposed to the last value only), time dependent CPT parameters for loss aversion and probability weighting, etc. Also, experiments about suitable models for multiple reference points and reference point adaptation and experiments to verify that typical CPT parameters are suitable choices also for the evaluation of future value fluctuations would be desirable.

## A Pricing formulae for the considered guaranteed products

In this Appendix we give the arbitrage free prices for the three guaranteed products considered in Section 3 at  $t_0 = 0$  and at each lock-in date  $t_1, \dots, t_{n-1}$ . The proofs of the pricing formulas of Proposition A.1 use similar techniques as presented by Ebert *et al.* (2012), who only gives prices for  $t = 0$ . We therefore omit the details.

**Proposition A.1** (Arbitrage free Pricing)

Let  $\mathcal{B}^{Put}(S_t, t, \theta, K, T)$  denote the time  $t$ -price of a European put option with underlying

$S$ , maturity  $T$  and strike  $K$ , i.e.

$$\mathcal{B}^{Put}(S_t, t, \theta, K, T) = e^{-r(T-t)} K \mathcal{N}\left(-h^{(2)}\left((T-t), \frac{S_t}{K}\right)\right) - S_t \mathcal{N}\left(-h^{(1)}\left((T-t), \frac{S_t}{K}\right)\right)$$

with

$$h^{(1)}(t, z) := \frac{\ln(z) + (r + \frac{1}{2}\theta^2\sigma^2)t}{\theta\sigma\sqrt{t}} \text{ and } h^{(2)}(t, z) := h^{(1)}(t, z) - \theta\sigma\sqrt{t}.$$

Here,  $\mathcal{N}(\cdot)$  denotes the one-dimensional and  $\mathcal{N}_d(\cdot)$  the  $d$ -dimensional cumulative standard normal distribution. Then the arbitrage free prices for  $m \in \{0, \dots, n-1\}$  are given by:

(i) Roll-up:

$$A_{t_m}^{rol}(g, \alpha, \theta) = \alpha V_{t_m} + \mathcal{B}^{Put}(\alpha V_{t_m}, t_m, \theta, e^{gT}, T)$$

(ii) Ratch-up:

$$\begin{aligned} A_{t_m}^{rat}(g, \alpha, \theta) &= \alpha V_{t_m} + \mathbf{1}_{\left\{\left\{V_{t_j} \leq \frac{e^{gT}}{\alpha}\right\}_{1 \leq j \leq m}\right\}} \left( e^{-r(T-t_m)+gT} \mathcal{N}_{n-m}(v_1, \overline{\Sigma}_m) \right. \\ &\quad \left. - \alpha V_{t_m} \mathcal{N}_{n-m}(v_2, \overline{\Sigma}_m) \right) + \sum_{i=1}^{n-1} I_{m,i} \end{aligned}$$

where

$$\begin{aligned} I_{m,i} &:= \mathbf{1}_{\{i \leq m\}} \mathbf{1}_{\left\{V_{t_i} \geq \frac{e^{gT}}{\alpha}, \left\{\frac{V_{t_j}}{V_{t_i}} \leq 1\right\}_{1 \leq j \leq m, j \neq i}\right\}} \alpha \mathcal{N}_{n-m}(v_3, \overline{\Sigma}_m) \left( e^{-r(T-t_m)} V_{t_i} - V_{t_m} \right) + \\ &\quad \mathbf{1}_{\{i > m\}} \alpha V_{t_m} \mathcal{N}_{i-m}(v_4, \Sigma_i^m) \left( e^{-r(T-t_i)} \mathcal{N}_{n-i}(v_5, \overline{\Sigma}_i) - \mathcal{N}_{n-i}(v_6, \overline{\Sigma}_i) \right) \end{aligned}$$

with

$$\begin{aligned}
 v_1 &:= \left( -h^{(2)} \left( t_{m+1} - t_m, \frac{\alpha V_{t_m}}{e^{gT}} \right), \dots, -h^{(2)} \left( T - t_m, \frac{\alpha V_{t_m}}{e^{gT}} \right) \right) \\
 v_2 &:= \left( -h^{(1)} \left( t_{m+1} - t_m, \frac{\alpha V_{t_m}}{e^{gT}} \right), \dots, -h^{(1)} \left( T - t_m, \frac{\alpha V_{t_m}}{e^{gT}} \right) \right) \\
 v_3 &:= \left( -h^{(1)} \left( t_{m+1} - t_m, \frac{V_{t_m}}{V_{t_i}} \right), \dots, -h^{(1)} \left( T - t_m, \frac{V_{t_m}}{V_{t_i}} \right) \right) \\
 v_4 &:= \left( h^{(1)} \left( t_i - t_m, \min \left( \frac{\alpha V_{t_m}}{e^{gT}}, \frac{V_{t_m}}{V_{t_1}}, \dots, \frac{V_{t_m}}{V_{t_m}} \right) \right), h^{(1)}(t_i - t_{m+1}, 1), \dots, \right. \\
 &\quad \left. h^{(1)}(t_i - t_{i-1}, 1) \right) \\
 v_5 &:= \left( -h^{(2)}(t_{i+1} - t_i, 1), \dots, -h^{(2)}(T - t_i, 1) \right) \\
 v_6 &:= \left( -h^{(1)}(t_{i+1} - t_i, 1), \dots, -h^{(1)}(T - t_i, 1) \right)
 \end{aligned}$$

and corresponding variance-covariance matrices:  $\Sigma_i, \Sigma_m^i, \overline{\Sigma}_m$ .

(iii) *Cliquet*:

$$A_{t_m}^{cli}(g, \alpha, \theta) = \left( \alpha^{\frac{1}{n}} + \mathcal{B}^{Put} \left( \alpha^{\frac{1}{n}}, 0, \theta, e^{g\Delta t}, \Delta t \right) \right)^{n-m} \prod_{i=1}^m \max \left( \alpha^{\frac{1}{n}} \frac{V_{t_i}}{V_{t_{i-1}}}, e^{g\Delta t} \right)$$

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